Simple Harmonic Motion

Summary and Practice

Definition 1: **SHM is a projection of uniform circular motion and is defined by only two quantities:**

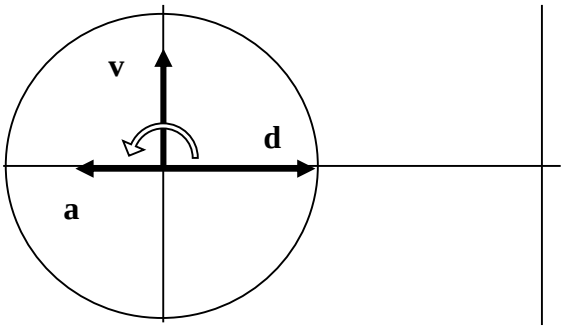
1. **Angular Frequency** ω or **Period** $T = \frac{2\pi}{\omega}$ or **frequency** $f = \frac{\omega}{2\pi}$
2. **Amplitude A**

Displacement: $\boxed{d = A\sin(\omega t)}$

Velocity: $\boxed{v = A\omega\cos(\omega t)}$ Velocity leads displacement by $\frac{\pi}{2} rad$

Acceleration: $\boxed{a = -A\omega^2\sin(\omega t)}$ Acceleration leads velocity by $\frac{\pi}{2} rad$

Phasor diagram



Maximum / Minimum	Central Position	Extreme Position
d	0	A
v	Aω	0
a	0	$A\omega^2$

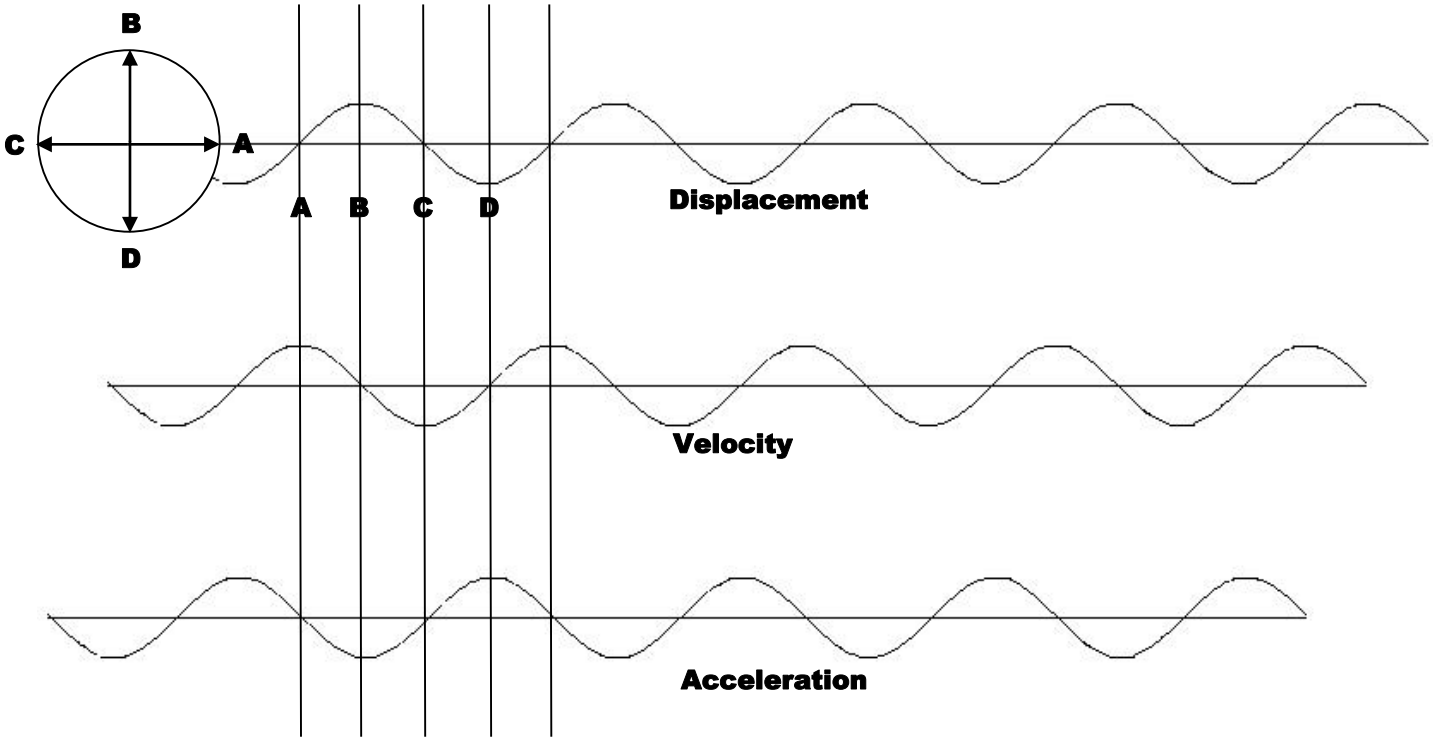
Definition 2: **Acceleration (and Force) is proportional to displacement and in opposite direction.**

restoring acceleration $a = -\omega^2 d$

restoring Force $F = ma = -m\omega^2 d$

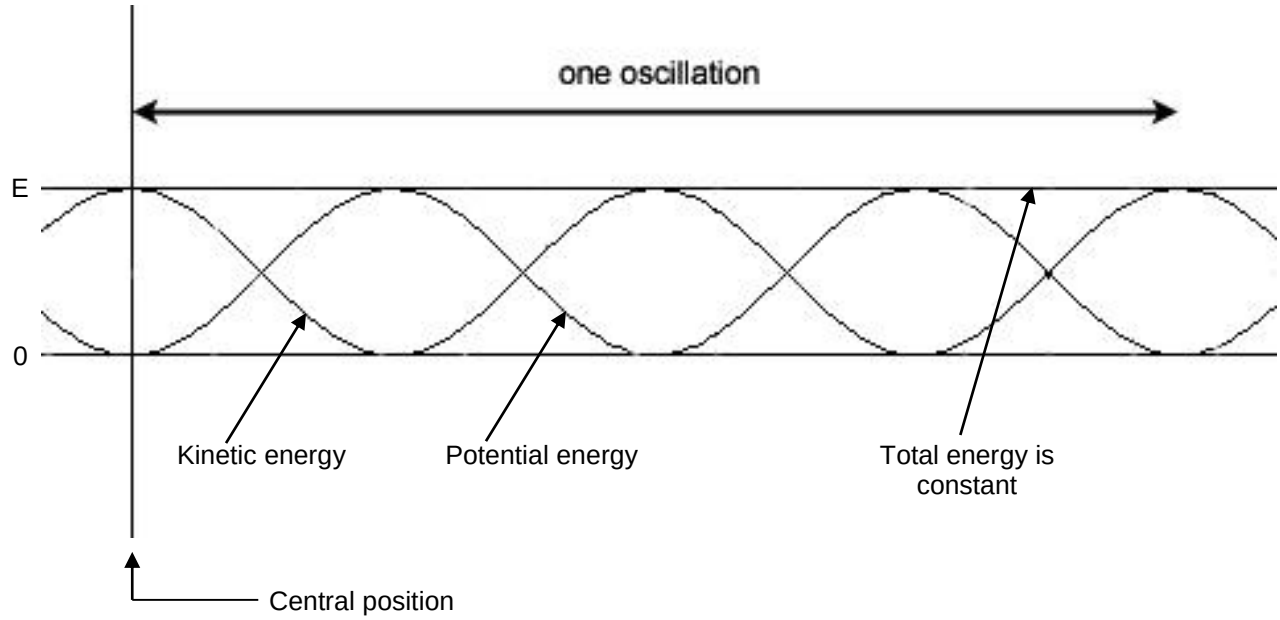
Choice of $t = 0$

There always is a phase difference of 90° between displacement, velocity and acceleration.
Check the consequence of $t = 0$ in the displacement/time graph
See diagram below (choice A, B, C or D).
This determines the type of function for d, v and a (± sin or ± cos).



Energy

There is a continuous exchange between potential and kinetic energy.
Potential energy can be gravitational and/or elastic.
Kinetic Energy is maximum when velocity is maximum (central position)
The sum of Potential and Kinetic Energy is constant.



Exercises

1 Which two quantities fully define a Simple Harmonic Motion?

2 Cross out and complete the following statements:

a Velocity leads / lags displacement by ….. degrees
b Acceleration leads / lags velocity by ….. ..degrees

3 A pendulum in a grandfathers clock is adjusted to a length of 2.0 m. The mass is pulled to one side (call that the positive direction) by 3.0 cm and released at t = 0. The pendulum swings form the start position to the opposite position in 1.0 s.

a What is the amplitude of the motion?

b What is the period (T) of the motion?

c Calculate the angular frequency.

d Sketch the displacement / time graph starting at t = 0, showing two complete oscillations. Label both axes.

e Write down the equation that describes the displacement as a function of time.

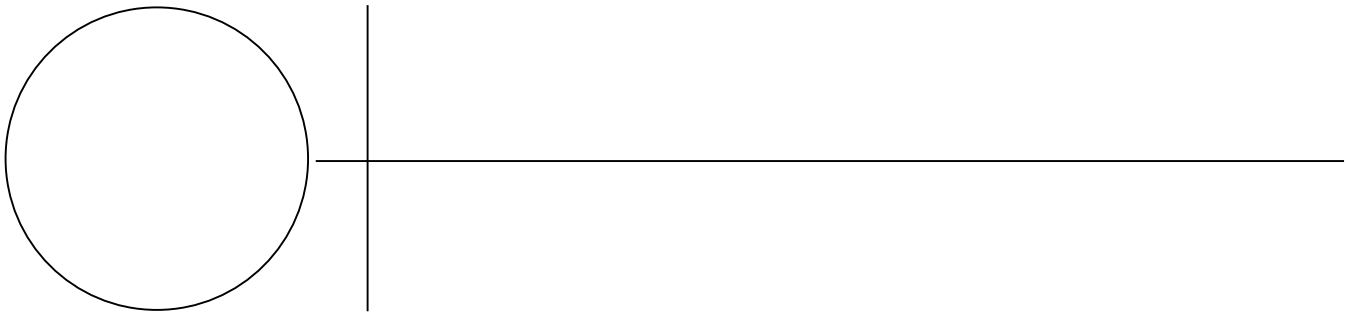
f Calculate the maximum velocity of the mass. At which position does this occur?

g Calculate the velocity of the mass (with proper sign) at t = 3.5 s.

4 A harbour experiences a tide with an amplitude of 2.3 m and a period of 12 hours.

a Calculate the period (in s) and the angular frequency (in rad s^{-1})

b Sketch a displacement / time graph with reference circle, spanning 24 hours.



c Use the reference circle to calculate for how long during each period the water level is 1.5 m above the average level.