Calculus Year 13 (Level 8)

Summary 4

Trigonometry

Radian measure

Radian defined such that $s = r \bullet \theta$ where s is the length along the arc of a circle with radius r and θ is the angle expressed in radians. Hence full circle gives $2\pi r = r \bullet \theta$ where $\theta = 2\pi$

Graphs of Trig Functions

Multiply function by a constant $y = 2\sin x$ alters AMPLITUDE

Multiplying angle by a constant $y = \sin 2x$ alters the PERIOD

Adding constant to function $y = \sin x + 2$ translates the graph vertically

Adding constant to the angle $y = \sin(x+1)$ translates the graph horizontally

Hint: play with GRAPHMATICA

Reciprocal Trig Functions

COSECANT $\cos ec\theta = \dfrac{1}{\sin\theta}$; SECANT $\sec\theta = \dfrac{1}{\cos\theta}$; COTANGENT $\cot\theta = \dfrac{1}{\tan\theta}$

Basic Identities

1. $\sin x \equiv \cos(\dfrac{\pi}{2} - x)$ and $\cos x \equiv \sin(\dfrac{\pi}{2} - x)$
2. $\dfrac{\sin x}{\cos x} \equiv \tan x$
3. $\sin^2 x + \cos^2 x \equiv 1$

Compound Angle Formulas

$\sin(A+B) \equiv \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) \equiv \sin A\cos B - \cos A\sin B$

$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$

$\tan(A+B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) \equiv \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Double Angle Formulas

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin_2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Sums and Products

$2\sin A\cos B \equiv \sin(A+B) + \sin(A-B)$ $\quad$ $2\cos A\sin B \equiv \sin(A+B) - \sin(A-B)$

$2\cos A\cos B \equiv \cos(A+B) + \cos(A-B)$ $\quad$ $2\sin A\sin B \equiv \cos(A-B) - \cos(A+B)$

Trigonometric Equations

INVERSE TRIG FUNCTIONS

Swap x and y in $y = f(x)$ (see Summary 2 Functions)

Range of inverse trig functions usually restricted to SET OF PRINCIPAL VALUES

Function	*Domain*	*Range (Principal Values)*
$y = \sin x$	$x \in R$	$[-1,1]$
$y = \cos x$	$x \in R$	$[-1,1]$
$y = \tan x$	$x \in R$	$y \in R$
$y = \sin^{-1} x$	$[-1,1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1,1]$	$[0,\pi]$
$y = \tan^{-1} x$	$x \in R$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

GENERAL SOLUTIONS

Function	*General Solution*
$k = \sin x$	$x = n\pi + (-1)^n \alpha$
$k = \cos x$	$x = 2n\pi \pm \alpha$
$k = \tan x$	$x = n\pi + \alpha$

Applications of Trigonometry

see Exercise 11.5