Calculus Year 13 (Level 8)

Summary 3

Geometry

Line

EQUATION $y = mx + c$ with gradient $m = \tan\theta$ where θ is the angle with the x-axis

ONE POINT (x_1, y_1) and GRADIENT m are given → $Y - y_1 = m(X - x_1)$

TWO POINTS $(x_1, y_1), (x_2, y_2)$ are given → $\dfrac{Y - y_1}{X - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

MIDPOINT of the line $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

ANGLE between two lines $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$ with special cases

PARALLEL if $m_1 = m_2$ hence $\tan\theta = 0$

PERPENDICULAR if $m_1 m_2 = 1$ or $m_1 = -\dfrac{1}{m_2}$ hence $\tan\theta$ is undefined

DISTANCE between two points (length of the line) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Locus

is set of all points (X, Y) satisfying a certain condition, e.g.

CIRCLE is locus of all points at distance r from the centre (a, b) hence $r = \sqrt{(X - a)^2 + (Y - b)^2}$

Translation

of a graph by vector $\begin{pmatrix} a \\ b \end{pmatrix}$ → substitute $x - a$ for x and $y - b$ for b

Example a circle through the origin $(0,0)$ is $r = \sqrt{X^2 + Y^2}$

when translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ becomes $r = \sqrt{(X - a)^2 + (Y - b)^2}$